Physics 101 - Lecture 12
Momentum & Collisions

Momentum is another quantity (like energy) that is tremendously useful because it's often conserved.

In fact, there are two conserved quantities that we can deal with: linear momentum, and rotational (angular) momentum.

The first of these, linear momentum, is particularly useful in analyzing collisions, such as those you'll see (have seen!) in Lab #3.

The momentum of an object of mass m is defined to be:

$$\boxed{\vec{p} = m\vec{v}}$$

where momentum p and velocity v are both vectors. The units of momentum are kg-m/sec (no special name) in the MKS system.

Note the difference with energy:

$E_{kin} = \frac{1}{2} mv^2$ scaler

$\vec{p} = m\vec{v}$ vector

Momentum and energy are related quantities, but not identical.

To change the momentum of an object requires the application of a force. In fact, Newton originally cast the second law F=ma as a statement about momentum:

> The net force on an object is equal to the rate of change of its momentum.

ie, $F = \Delta p / \Delta t = \Delta(mv)/\Delta t$

We see $\Delta p = m\Delta v$ mass = constant

so $F = m\, \Delta v/\Delta t = ma$ (as usual !)

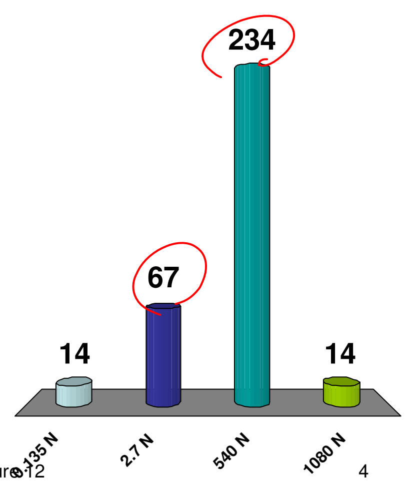
However, note that

$$F = \Delta p / \Delta t$$

is more general; for example, there can be cases where the mass might change (eg. rocket engines).

Use momentum to calculate the force on a well-hit golf ball (m=45 g) as it is struck, moving from rest to 60 m/s in 5 ms.

1. 0.135 N
2. 2.7 N
3. 540 N
4. 1080 N



Example: Use momentum to calculate the force on a well-hit golf ball (m=45 g) as it is struck, moving from rest to 60 m/s in 5 ms.

$F = \Delta p / \Delta t$

$= \Delta (mv) / \Delta t$

$= m \Delta v / \Delta t$

$= m(v_f - v_i) / \Delta t$

$= (0.045)\,kg \cdot (60\ m/s) / 0.005\ sec$

$\underbrace{F = 540\ N}$

$m_i v_i = 0 \rightarrow m_f v_f$

kinematics

or Δp

Two objects have the same kinetic energy. How do the magnitudes of their momenta compare?

1. $p_1 = p_2$
2. ~~2.~~ $p_1 < p_2$
3. ~~3.~~ $p_1 > p_2$
4. Not enough information to tell

$E_1 = E_2$

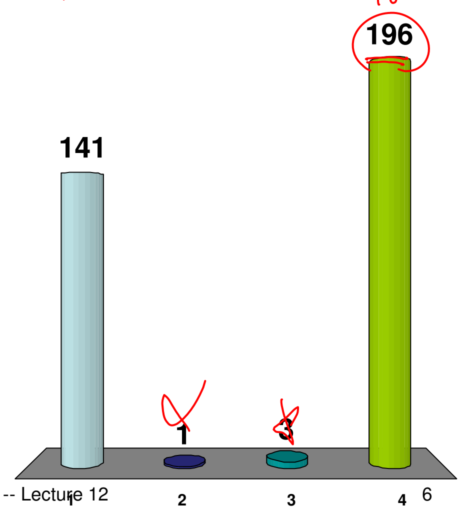
$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$

$p_1 \ ? \ p_2$

$m_1 v_1 \quad m_2 v_2$

201

146



A car (m) and a truck (M) travelling at the same speed are in a head-on collision, and stick together. Which vehicle experiences a larger change in the magnitude of momentum?

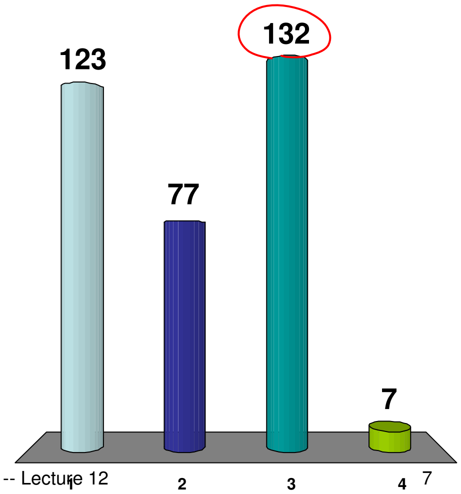
$m < M$

1. ~~The car~~
2. ~~The truck~~
3. They have the same change in the magnitude of p
4. Not enough information to tell

$|F_1| = |F_2|$

$F = \frac{\Delta p}{\Delta t}$

$|\Delta p_1| = |\Delta p_2|$

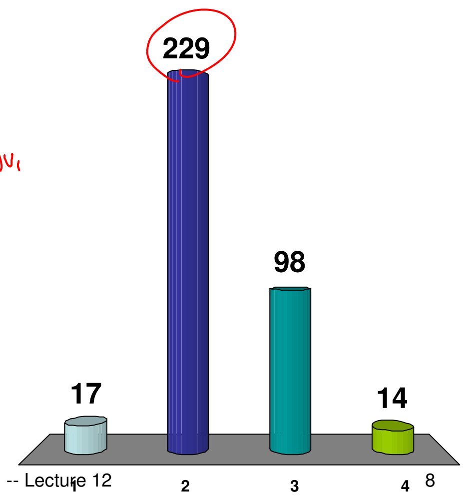


A friend throws a baseball (m) to you, and you catch it. Then (s)he throws a medicine ball (M) (several kg) to you. Which is easier to catch?

$M > m$

1. A medicine ball thrown at the same speed as the baseball
2. A medicine ball thrown at the same momentum as the baseball $m v_1 = M v_2$ $v_2 = \left(\frac{m}{M}\right) v_1$
3. A medicine ball thrown at the same kinetic energy as the baseball $\frac{1}{2} m v_1^2 = \frac{1}{2} M v_2^2$
4. I don't know – I'd duck anyways!

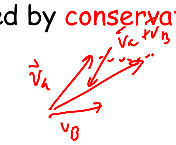
$v_2 = \left(\frac{m}{M}\right)^{1/2} v_1$



Conservation of momentum

One of the great uses of momentum is to understand the dynamics of collisions, which are characterized by conservation of momentum:

vector quantity



Consider particles A (mass m_A, velocity v_A) and B (m_B, v_B). Then in a collision:

before collision

$$\boxed{m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A{}' + m_B\vec{v}_B{}'}$$

after collision

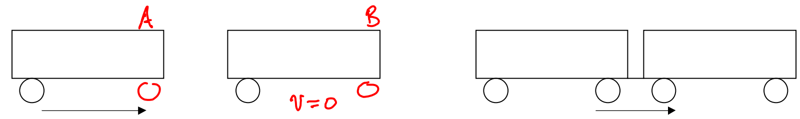
where this is to be understood as a vector equation and the primed quantities are after the collision.

In general:

The total momentum of an isolated system remains constant.

You can show (see Giancoli) that this equation can be derived from Newton's laws – that is, there is nothing new here, just a new way to think about it.

Example: Giancoli #7.8. A 9300 kg boxcar moving 15.0 m/s strikes and sticks to a boxcar at rest. After the collision, the two move at 6 m/s. What is the second boxcar's mass? $\longrightarrow$ 1D.



9300 kg, 15 m/s

v_1'=6 m/s, M=9300+m_2

$\vec{p}_{init} = m_A\vec{v}_A + m_B\cancel{\vec{v}_B}$ (0)

$\vec{p}_f = (m_A + m_B)\vec{v}_f'$

$p_i = m_A v_A$
$= (9300)(15)$ kg m/s

$p_f = (9300 + m_B) \cdot 6$ kg m/s

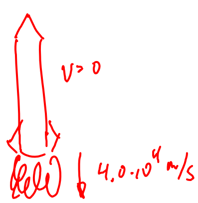
$p_i = p_f$

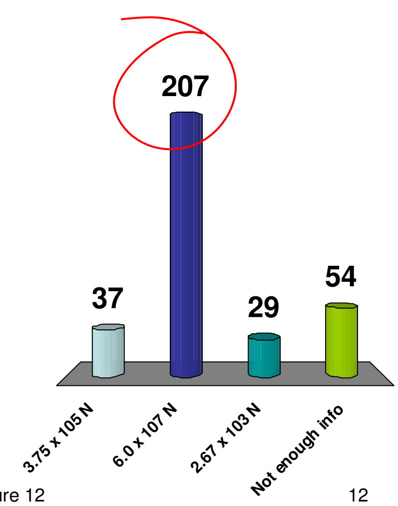
$m_B = 9300 \cdot 15/6 - 9300$ kg

$\boxed{m_B = 13950 \text{ kg}}$

Giancoli #7.5. What is the force exerted on a rocket, if the propelling gases are expelled at the rate of 1500 kg/s with a speed of 4.0x10^4 m/s at liftoff?

1. 3.75 x 10^5 N
2. 6.0 x 10^7 N
3. 2.67 x 10^3 N
4. Not enough info





Example: Giancoli #7.5. What is the force exerted on a rocket, if the propelling gases are expelled at the rate of 1500 kg/s with a speed of 4.0×10^4 m/s at liftoff?

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t}$$

$$= v \boxed{\frac{\Delta m}{\Delta t}} \quad \text{mass change rate}$$

$$F_{rock} = 4 \cdot 10^4 \, m/s \cdot 1500 \, kg/s$$

$$\boxed{F = 6 \cdot 10^7 \, N}$$

Collisions

Collisions are an essential part of physics, and usually involve large forces acting over short times.

From Newton's second law, the net force on an object is related to the change in momentum:

$$F = \Delta p / \Delta t$$

or $$F \Delta t = \Delta p$$

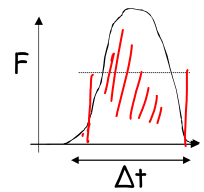
This is called the impulse = F Δt ; $= \Delta p$

So

impulse = F Δt = Δp = change of momentum

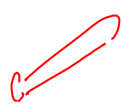
This concept of 'impulse' is often used for forces acting for a very short time period. Although the force may vary during this time we can make the approximation of a constant force over the interval:

$$\Delta p = F\Delta t$$



Example: What impulse[1] is experienced by a baseball (mass 150 g, incoming speed v=35 m/s) as it hits a bat and moves off at v=45 m/s? If the ball-bat contact time is 2 ms, what is the average force[2] applied?

+ve



$v_i = -35$ m/s

$v_f = +45$ m/s

$\Delta t = 0.002$ sec

Impulse : $F\Delta t = \Delta p$

$\Delta p = p_f - p_i = m(\vec{v}_f - \vec{v}_i)$

$= (0.150\text{ kg})(45 - {-35})$ m/s $= 0.150 \cdot 80$ kg m/s

$= 12.0$ kg m/s

Avg Force : $F_{avg} = \frac{\Delta p}{\Delta t} = \frac{12.0}{0.002}$ N

$= 6 \cdot 10^3$ N

Conservation of Energy

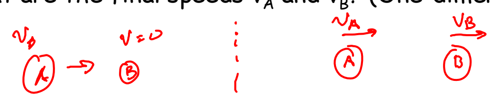
We saw that momentum is conserved. In addition, we will often find that the kinetic energy is conserved; collisions of this type are called elastic collisions.

Note: Not all collisions are elastic; in inelastic collisions, the total energy is conserved but not the kinetic energy; other types of energy (heat, deformation, etc…) come into play. In perfectly (or completely) inelastic collisions, the two objects stick together after the collision.

For elastic collisions, we have two equations relating dynamical variables:

$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$	cons. of momentum
$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$	cons. of energy (elastic)

Example: Billiard balls: ball A collides at speed v with B at rest. IF the collision is elastic, what are the final speeds v_A and v_B? (One-dimensional problem !).



Mom: $\cancel{m}v + 0 = \cancel{m}v_A + \cancel{m}v_B \rightarrow V = V_A + V_B$

E : $\frac{1}{2}\cancel{m}v^2 + 0 = \frac{1}{2}\cancel{m}v_A^2 + \frac{1}{2}\cancel{m}v_B^2$

$v^2 = v_A^2 + v_B^2$

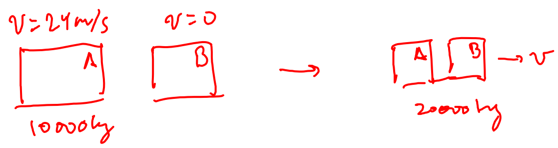
$(V_A + V_B)^2 = V_A^2 + V_B^2$

$\cancel{V_A^2} + 2V_A V_B + \cancel{V_B^2} = \cancel{V_A^2} + \cancel{V_B^2}$

$2 V_A V_B = 0$ $\quad V_A = 0$ or $\cancel{V_B = 0}$

$\Rightarrow V_B = V$

Example #2: Giancoli example 7-9: two railroad cars, A & B (10,000 kg each). A, at 24.0 m/s, strikes B at rest and they lock together. What is the final speed v, and how much kinetic energy was lost in the collision?



p: $p_i = p_f$ $\quad \downarrow p_B$

$(24 m/s)(10000 kg) + 0 = (20000 kg)\, v$

$v = 12\ m/s, = v_A/2$

E: $\frac{1}{2} m_A v_A^2 = E_{init} = \frac{1}{2} m v_A^2$

$E_f = \frac{1}{2}(m_A + m_B) v^2 = \frac{1}{2}(2m)\cdot\left(\frac{v_A}{2}\right)^2 = \frac{1}{4} m v_A^2$

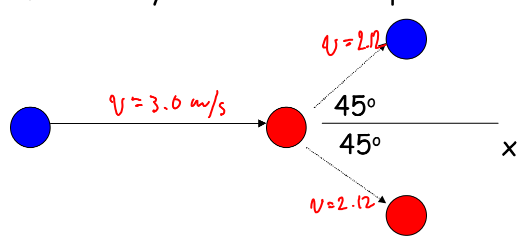
$\neq$

$\boxed{50\%\ \text{of}\ E_{kin}\ \text{lost}}$

Note that our examples so far have been 1-dimensional, but the momentum conservation law is a vector equation. So we can do these same problems in 2- or 3-dimensions.

Example #3: Giancoli example 7-11: two billiard balls colliding in 2-D. Ball A (v=3.0 m/s) collides with B (at rest), and the two move off at 45° to the direction of A initially. What are their speeds ?

Solution:



Firstly note that from symmetry, the two speeds must be the same: otherwise the component of p perpendicular to the axis x will not cancel.

When we balance the x components of momentum we get an equation in the final speeds v:

$$mv_A + 0 = mv\,(\cos 45^o) + mv\,(\cos 45^o)$$

So $v = v_A/(2\cos 45^o)$
$= 3.0\ \mathrm{m/s}\,/(2 \times 0.707)$
$= 2.12\ \mathrm{m/s}$

Is the collision elastic ? The energy equation is:

$$\tfrac{1}{2}\,mv_A^2 \overset{?}{=} 2 \times \tfrac{1}{2}\,m\,v^2 + E_{lost}$$
$$E_{lost} = \tfrac{1}{2}\,mv_A^2 - mv^2 = m\,(\,(0.5)\,(3.0)^2 - 2.12^2)$$
$$= 0$$

So energy was conserved; the collision was elastic.

The story so far:

- o momentum: $\vec{p} = m\vec{v}$
- o impulse = $\vec{F}\Delta t = \Delta\vec{p}$

- o momentum is conserved in collisions

- o in elastic collisions, energy is conserved
- o in inelastic collisions, energy is not conserved
  (in perfectly inelastic collisions, colliding objects stick together)

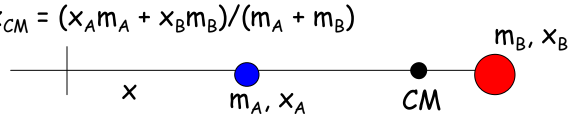
Up until now we have either considered point objects, or made the approximation that our extended objects can be considered as point masses.

In fact, real macroscopic objects have a spatial extent and can undergo other motions than simple translational motion: they can rotate, deform, etc…

We talk about the center of mass (CM) of an object, and its motion. The center of mass is the weighted position (ie, the mass-averaged position) of all the masses making up a body:

For example, for two masses m_A and m_B, then the position of the CM will be between them at a position:

$$x_{CM} = (x_A m_A + x_B m_B)/(m_A + m_B)$$



The center of mass is always closer to the larger masses, in 2-body cases.

Example: The distance from the Earth to the Moon is $3.85\text{x}10^5$ km. The Earth is 81 times more massive than the Moon; where is the CM of the Earth-Moon system ?

$$x_{CM} = (x_E m_E + x_M m_M)/(m_E + m_M)$$

Take the zero of our scale to be the center of the Earth, so we get:

$$x_{CM} = x_M m_M/(m_E + m_M) = x_M/(1 + m_E/m_M)$$
$$= 3.85\text{x}10^5 \text{ km} /(1 + 81) = 4700 \text{ km}$$

That is, the center of the system is 4700 km from the center of the Earth, or about 1700 km under the Earth's surface.

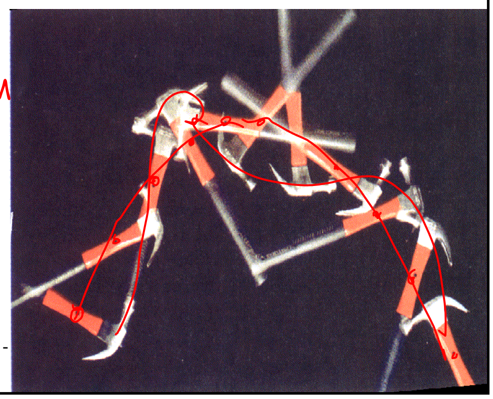
An important reason to talk about the center of mass is that the dynamics of a complicated body or system can be broken down into:

- the motion of the center of mass (more later)

plus - internal motions such as rotations, vibrations, etc…

As an example, if a hammer is spun while it is thrown into the air, the motion looks complicated…

BUT… it is really free-fall of the CM together with the twisting, etc… "internal" to the system.



Example: Giancoli 7-27. Two bumper cars A and B (masses: 450 and 550 kg, respectively) are at 4.50 m/s and 3.7 m/s in the same direction. Car A bumps B from behind in an elastic collision. What are the final speeds ?

A → B →

450 kg, 4.5 m/s 550 kg, 3.7 m/s

1st method: use conservation of momentum:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

to express v_B' in terms of v_A':

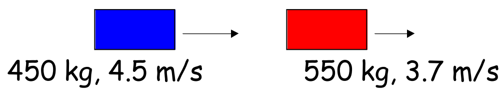
$$v_B' = (m_A v_A + m_B v_B - m_A v_A')/m_B$$

And now plug this expression for v_B' into the energy equation:

$$\tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 = \tfrac{1}{2} m_A v_A'^2 + \tfrac{1}{2} m_B v_B'^2$$

and solve the resulting quadratic for v_A'.

Second (easier) method:



Start by calculating the CM, and its motion. It will always be between the two (closer to the 550 kg car), and its velocity will be:

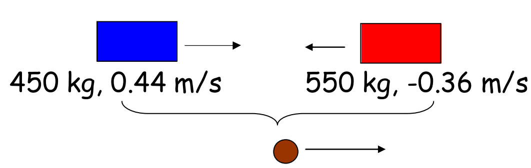
$v_{CM} = (m_A v_A + m_B v_B)/(m_A + m_B) = (450x4.5 + 550x3.7)/1000$

$v_{CM} = 4.06$ m/s (to the right)

Now we calculate the speeds of the two cars <u>in the CM</u>:

$v_{A,CM} = v_A - v_{CM} = 4.50 - 4.06 = 0.44$ m/s (right)

$v_{B,CM} = v_B - v_{CM} = 3.70 - 4.06 = -0.36$ m/s (left)



CM

In the (moving) center of mass, the total momentum initially is:

$m_A v_{A,CM} + m_B v_{B,CM} = 450 \times 0.44 - 550 \times 0.36 = 0$!!! (why?)

So the final momentum is:

$0 = m_A v_{A,CM}' + m_B v_{B,CM}'$

ie $v_{A,CM}' = -(m_B/m_A) v_{B,CM}'$

Now the conservation of energy equation says gives us

$E_{init} = \frac{1}{2} m_A v_{A,CM}^2 + \frac{1}{2} m_B v_{B,CM}^2 = 79.2$ J

So $\frac{1}{2} m_A v_{A,CM}'^2 + \frac{1}{2} m_B v_{B,CM}'^2 = \frac{1}{2} ((m_B^2/m_A) + m_B) v_{B,CM}'^2 = 79.2$ J

Solving for v_B':

$v_B'^2 = 79.2 \times 2/((m_B^2/m_A)+m_B)$

$v_B' = 0.36$ m/s

and $v_A' = (550/450)\ 0.36 = -0.44$ m/s

But these are the velocities with respect to the CM, so the final (lab) velocities are:

$v_A' = v_{CM} + v_{A,CM}' = 4.06$ m/s $- 0.44$ m/s $= 3.62$ m/s

$v_B' = v_{CM} + v_{B,CM}' = 4.06 + 0.36 = 4.42$ m/s

That's the end of the question, but let's go back to an intermediate result to generalize it.

Recall that we had:

$v_{A,CM} = 0.44$ m/s

$v_{B,CM} = -0.36$ m/s

And $v_{A,CM}' = -0.44$ m/s

$v_{B,CM}' = 0.36$ m/s

Note that:

$v_A - v_B = -(v_A' - v_B')$

$v_A - v_B = -(v_A' - v_B')$ (1-D elastic collisions)

In words: the relative speed of the two objects before the collision is the same (in magnitude) as after the collision.

This is a general result for head-on elastic collisions – that is, collisions in one dimension. (See Giancoli 7-5 for a derivation).

Assignment for the next lecture:

Read Chapter 8

Questions, problems from all sections of Chapter 7